Introduction

This paper presents mostly qualitative data gathered during three non-sequential semesters in a graduate school of education. Our goal was to explore prospective teachers’ beliefs about early childhood education and the dilemmas they confront. We begin with an overview of the need for mathematics at the preschool and kindergarten levels and the obstacles encountered in trying to fulfill this need: the role of teacher beliefs in their instructional decisions, as well as the current state of teacher education programs. We then discuss a course at Teachers College, Columbia University whose aim is to educate early childhood education (ECE) teachers on children’s mathematical thinking, pedagogy, curriculum, mathematics, and assessment. Weekly student reflections and some other data serve as the basis for our discussion of ECE teachers’ beliefs about early childhood math education, their expectancies concerning children’s abilities, their fears and views of the obstacles facing them, their feelings about their own ability to do math and to teach it to young children.

The Rand Mathematics Study Group recommends a “research agenda to investigate what mathematical knowledge is needed for teachers and the means necessary to help teachers acquire, apply, and use this knowledge” (Bowman et al., 2001). To promote change in ECE teacher training, NCTM and NAEYC published a statement (NAEYC & NCTM, 2002) illuminating the need for high quality, challenging, and accessible mathematics education for children aged 3 to 6. Within this report recommendations were made regarding the specific domains children should be introduced to: operations, algebra, geometry, measurement, data analysis, and probability. In order for our teachers to disseminate this knowledge, however, they must first be properly trained.

The NRC (2001) recommends that teachers of 3- to 5-year-olds hold a BA with a specialization in early childhood education. However, the 2004 National Institute for Early Education Research Yearbook reports that 23 of the 44 state-financed preschool
initiatives profiled required lead teachers to hold a BA degree only when teaching in a public school (Ginsburg, 2010). Unfortunately, preservice ECE teachers rarely receive appropriate mathematics education training even when enrolled in higher education programs (Clements et al., 2001; Darling-Hammond, 1997; Copley and Padron, 1999). This means that teachers entering the classroom have generally received the least amount of training in the subject with which they need the most help. Much to our chagrin, we know that once teachers are in the classroom they continue to get very little in-service training (Ginsburg, 2010). Said concisely by Brown, “a lack of appropriate knowledge and preparation could cause teacher candidates and experienced teachers to fail to see mathematics as a priority for young children and have less confidence in their ability to teach mathematics effectively” (2005).

Demographics

There are currently 2.3 million people in the ECE workforce (Ginsburg, 2010). 48% of caregivers are either a relative, friend, or neighbor. Only 11% of that group has earned a BA or greater. Of the remaining members of the ECE workforce, 24% work in childcare centers, like Head Start. Head Start teachers tend to have more experience in terms of formal education: 36% of them have acquired at least a BA. Research has shown that ECE teachers with higher degrees of education provide higher quality educational environments for their students, which ultimately lead to better outcomes for the students (Barnett, 2003; Bowman, et al., 2001; Dunn, 1993; Helburn 1995, as cited in Brown, 2005).

Another setback for ECE teachers and those interested in the field is the lack of compensation. The average preschool teacher salary is $25,800. A childcare worker salary is $19,760, while a Head Start teacher’s salary is $23,608. In addition, many are not provided with health insurance (Ginsburg, 2010). Given these numbers, there is not much incentive for people to enter the field, nor for people to stay in it long. As for getting a higher education degree, the chances are slim they can even afford it.

Call for more math education

Despite these statistics and the current state of preservice teacher education, national and state-level initiatives to incorporate mathematics education into early childhood curricula are in full swing. Implementing early childhood education programs stems partly from concern that American students are not performing as well as they should on nationwide and international comparisons, especially given the new technologies in our society and the mathematical knowledge required to maintain them (NRC, 2001). Research has shown, however, that the gap between American students’ abilities and those of children in China, Japan, and Korea is not terribly wide at the age of four (Case et al., 1996; Ginsburg, 1997). However, the gap widens in kindergarten and grows even larger by the fourth grade level (DOE & NCES, 1997). Why is this the case? Perhaps it is tied to the fact that ECE teachers receive very little compensation for their work and very few of them have received proper education and training. We cannot expect effective teaching in the classroom under these circumstances.

Although recent research suggests young children’s potential for mathematical
thinking, preservice and in-service teachers tend to underestimate their students’ abilities. Young children are ready, willing, and capable of learning abstract concepts (Ginsburg, 2010). “Young children see mathematics as meaningful, interesting, and worth learning. The desire for young learners to quantify in their world is natural. They have the disposition and the motivation to quantify the world around them (NRC, 2001; Ginsburg & Amit, 2008). “The challenge for teachers is to build on the children’s initial, fragile mathematical understandings to make them more reliable” (Brown, 2005, p. 243).

Unfortunately, many teachers enter the workforce ill prepared, allowing their own beliefs about children’s learning to interfere with their instruction. “The instructional climate of prekindergarten classrooms may inadvertently be a contributor in the ever-growing gap in mathematics achievement between European Americans and both African Americans and children of poverty” (2005, p. 244).

With new standards for ECE teachers from NAEYC (2003), there is an increased need to adjust our focus on teacher preparation courses. It is up to us to help them become good teachers. Research shows that the level of preparation provided to the typical ECE teacher is far below what is considered optimal for children’s learning and development (Bowman et al., 2001). If our preschool and kindergarten students are not provided with adequate foundations for understanding mathematical concepts, then they are less likely to pick up the more abstract mathematical ideas introduced in the later grades (Strauss et al., 1998). Thus, the gap is growing wide by fourth grade.

What teachers should know

NAEYC guidelines for ECE teacher preparation speak to the importance of being able to recognize the concepts and skills that are developmentally appropriate for young children. More importantly, they must know “not only what is important in each content area but also why it is important—how it links with earlier and later understandings both within and across areas” (NAEYC, 2003). Learning how children’s cognitive, language, social and emotional development impact their understanding of each topic needs to be a critical component, as research has shown those elements to be important to child development (2003). Thus, with the call for curricular mathematics at the preschool and kindergarten levels comes the need for a change in preservice teacher training. ECE teachers need to be knowledgeable enough to be flexible and able to adapt their lessons to their students’ level of understanding.

This implies that teachers need to know what is and is not developmentally appropriate mathematics, how to identify and understand what a student knows about math content, and consequently how to mold their own instruction to scaffold the child’s knowledge. Teachers need to know the basics of how young children learn. They also need to be able to implement lesson plans that speak to the diverse ways in which they can learn.

How beliefs/memories affect what teachers thinking

Evidence suggests that ECE teachers’ memories of their own schooling impact their motivations, expectations, and values in their classrooms (Hollingsworth, 1989; Nespor, 1987). Ball and Cohen (1996) note that “teachers have implicit beliefs about
subject matter, their students, and their roles and responsibilities that significantly influence how they behave in their classroom” (p. 6); and Thompson (1984) concludes that “there is strong reason to believe that in mathematics, teachers’ conceptions (their beliefs, views, and preferences) about the subject matter and its teaching play an important role in affecting their effectiveness as the primary mediators between the subject and the learners” (p. 105). Beliefs generally materialize as attitudes, judgments, or perspectives (Strauss, 1993). Additionally, beliefs tend to be stable and quite resistant to change (Brousseau, Book, & Bryers, 1988). Given that beliefs held by teachers regulate their perceptions, judgments, and behaviors in the classroom (Bandura, 1986), understanding teacher belief structures is central to improving their preparation and classroom practices (Pajares, 1992). For example, a recent study found that female teachers’ math anxieties can be an impediment to their own math achievement. More importantly, girls with “math anxious” teachers were more likely to endorse the stereotype that boys are good at math and girls are not by the end of the school year (Beilock et al., 2009).

Beliefs about how children learn

Research has shown that teacher’s mental models of how children’s minds work tend to be very similar, even when taking into account the teacher’s subject matter knowledge and years of experience (Strauss, 1998). Many teachers view children as being concrete learners; it is widely accepted that children cannot think or understand abstract ideas (Strauss, 1993). The general perception is that teachers possess “the knowledge” and it is their job to get that into the children’s minds. Thus, teachers will have beliefs about how the knowledge can get inside, as well as beliefs about how it can be stored once it is inside (1993). A widely accepted theory is that children’s minds have small, flap-like openings. A “good” teacher is often thought of as someone who can break the information into chunks small enough to fit through the openings (1993). Once the information has traveled through these little openings, many believe the child links the new information to previously stored information. Therefore, in the absence of existing knowledge, many teachers think the only way for the child to store new information is through repetition and practice (1993). What is remarkable is that many teachers possess a similar mental model, yet this model for learning is never taught to them: the models are constructed independent of preservice and in-service training.

In possessing these mental models, teachers make assumptions about children’s cognitive processes: children have already-learned knowledge and this knowledge can be retrieved, generalized, and then connected to new knowledge. These assumptions not only affect the teachers’ instructional goals but also the cognitive goals they want their students to achieve. “There is strong reason to believe that teachers’ beliefs and conceptions about mathematics and teaching mathematics play a vital role in their effectiveness as mediators between the subject and the learner” (Lindgren, 1996; Ma, 1999; National Research Council, 2001; Thompson, 1984, as cited in Brown, 2005). These pre-conceived notions of how children learn often lead teachers’ expectancies to be misaligned students’ actual abilities (Strauss, 1993).

Relying on teaching reforms will be unsuccessful unless we find a way to change teachers’ deeply held beliefs about the teaching and learning of mathematics.
Historically, very little research has been conducted on the education of teachers, their teaching practice, and its effect on their students (Ginsburg & Amit, 2008). One study, however did examine preschool mathematics teaching. The results of this study, however, are indicative of the state of the ECE workforce: during 12 hours of observation in 2 classrooms, researchers found only 12 instances of mathematical discussion and only 3 instances of direct teaching (Graham, Nash, & Paul 1997, p.35, as cited by Ginsburg & Amit, 2008).

The more we learn about the development of teachers’ teaching practice from their coursework and student teaching experiences,

... the more we are able to construct models or theories of professional growth that will be able to shape the construction of future courses, inform the training and induction of teachers, and serve as guides for action for teacher educators dealing with the complex task of helping student teachers to learn the practice of teaching (Calderhead & Shorrock, p. 9, cited by Akinsola, 2009).

How do we mobilize teachers to reassess their understanding of their students given these conditions? Consider next our approach involving a course on the Development of Mathematical Thinking.

Our Course

In an attempt to change the direction of ECE teacher training, Dr. Herbert Ginsburg started The Development of Mathematical Thinking (DMT) course at Teachers College, Columbia University in the fall of 2004. The course provides a rich offering of the mathematical content preschool and kindergarten teachers should be introduced to help their students understand the math and use it in their daily life. The course delivers the type of content a high-quality ECE teacher should be able to understand: basic mathematical ideas, children’s everyday learning and thinking, how to assess children’s learning and thinking, how to develop appropriate pedagogy for children, and how to master a mathematics curriculum (Ginsburg et al., 2005). The content is delivered through readings, watching and analyzing videos of adults interviewing children engaged in various mathematical behaviors, learning how to conduct a Clinical Interview (CI), and subsequently conducting an interview with a child on their own.

One of the course goals is to create awareness of young children’s capabilities in learning mathematical concepts. To accomplish this, teachers need to reflect on their own theories of how children learn mathematics so they can integrate new information being taught in the course into those theories. Observations and interpretations communicated through online and in-class discussions are at the core of this concept. Through discussion, the teachers develop awareness of their personal theories of mathematics learning and their beliefs about children’s math abilities (Ginsburg et al., 2005).

A second goal of the course is to empower teachers to select appropriate activities for their students. This involves truly understanding the mathematics being taught to the children (MA, 1999) and developing a deeper understanding of how the teacher can serve as a guide for the students in delivering instruction (Vygotsky, 1978).

A third goal of the course is to learn how to assess the development of mathematical thinking in the children and the children’s knowledge of math. This means the teachers need to be able to assess the effects of their instruction through quantitative
evaluations and informal assessments (Ginsburg et al., 1998). These goals are largely achieved through learning to conduct and critique a CI. Through observation, interpretation, and asking the right questions at the right time, learning to conduct a CI aims to show teachers they can gain access to children’s thinking.

Developed by Piaget in 1976, the clinical interview (CI) is a method through which a child’s thinking can be explored. At its core, the CI is an opportunity for the adult to test an hypothesis regarding the child’s understanding of a particular concept. It is unique in that no interview is alike as each is composed of “individualized patterns of questioning” (Ginsburg p.166). The interviewer begins with a protocol, which is based on the original hypothesis, but has the freedom to alter the tasks as the interview progresses. This flexibility comes also from the child’s partial control of the interview (Ginsburg, p.39). This is something that sits at the core of the concept of CI. These revisions are evidenced by on-the-spot creations of new tasks in response to the child’s verbalizations. For example, a child can point to and say shape names when presented with a prototypical image of the shape. Instead of concluding the child knows his shapes, the researcher then asks the child to find a shape based on its attributes, “please give me a shape with three sides.” In doing this, the teacher is able to determine where in the child’s conception of shapes wanes.

The course takes advantage of its access to an online platform, VITAL. In part it provides a “safe” place for the students to be reflective of their own beliefs and self-efficacy regarding learning and teaching math. They are asked to submit a reflection within 24-hours of each class session. The reflections, which are only visible to the professor, permit students to openly share anything on their mind with regards to the class. They are read each week by the professor and privately responded to. Typically, the most salient ones are selected to share at the beginning of the following class session. Completing these reflections evokes a lot of emotion in many students. They are used as a springboard for class discussion and often lead into the next class topic. The reflections enforce the idea that each person’s experience with math and with learning is different; people have memories and opinions on a whole range of content that is personal and quite unique. We use the reflections to open our eyes and help us understand the next generation of ECE teachers; the students’ experiences with math are quite emotional and typically guide what they are able and/or willing to glean from the course. Thus, they are important for the professor because they provide insight into how the students are integrating the information from the course with their own thoughts.

Our study

Method

To explore the nature of the preservice teachers’ learning experience, we gathered reflections from 3 semesters of the course: Fall 2006 (158 reflections), Fall 2007 (143 reflections), and Fall 2009 (162 reflections). These semesters were selected because of the similarity in course content and because the student population was, for the most part, homogenous. The typical student during these 3 semesters was enrolled in an early childhood education Masters program at Teachers College, Columbia University. While we do not have the specific demographics from of our population, we do know that 90%
of teachers are female (Beilock et al., 2009) and almost all students in the class were women. Based on their reflections, we can estimate about 15% of the students spent a portion of their life living outside of the United States.

We chose to examine reflections from four specific course sessions: salient math memories (127 reflections), math is all around us/play (122 reflections), transition to symbols/school number (109 reflections), and curriculum (105 reflections). Each of these topics serves as a turning point in the context of the course and in the development of children’s mathematical thinking. The recommended length for each reflection is 2-3 sentences; however students are instructed to write freely. Inevitably, some reflections are just a few words long while others are several paragraphs. The students are made aware that they receive full credit for just having submitted the reflection, it is not graded; this eases the pressure allowing them to talk more openly in their reflections.

The first reflection we chose to look at is from the first session of the class; students are asked to share their most salient math memory. In the following course session, students are introduced to the idea that “math is all around us” and that children engage in mathematical thinking prior to starting formal schooling (the basis for reflection 2). A few weeks later, the course looks at the transition from informal mathematics knowledge to linking that knowledge to more formal topics such as number sense and understanding symbols (the basis for reflection 3). The final “turn” in the course comes when the class discusses various types of curricula currently being implemented (the basis for reflection 4). In sum, the entire reflection analysis is based on 463 student reflections.

**Reflection 1: Salient Math Memories**

In reading the salient math memory reflections, several important issues emerged. We found that students have strongly held convictions about their experience with math with regards to content, the impression of teachers or parents, the effects of their abilities on their self-esteem/ self-efficacy, and their competitive drive. Whether positive or negative, the students’ opinions of their math memories we strongly held. We believe these emotions and memories play a role in the students’ current self-efficacy towards math.

Of the 127 reflections, we found that 36 students had strongly positive emotions towards their overall early-math learning.

I might be one of the few people that will ever say this but I LOVE math! Math has always been the subject I was best at and at one point I considered being a math teacher. My best memory of math is when my AP calculus class started to work on how to find a derivation of an equation ... what a great feeling!"

For some students, the experience was equally positive, but originated from their environment not from themselves. For the student below, the school’s teaching philosophy impacted the student’s experience with math:

Unlike the majority of people in the class, I had a great experience with math in my early childhood/elementary years. The school I went to promoted a philosophy of discovery and discouraged any rote memorization. My most salient memory of early math is using tangrams to make designs and patterns. I loved [the] work and I was not aware that I was doing any sort of
Not all students responded as enthusiastically when asked the question, however; 31 of the students expressed negative emotions towards their early math learning. One student wrote, “My experience with learning math has always been frightening. I guess ... a subject that is challenging for a person can bring along fear.”

Among the 127 students who responded, 18 spoke of a shift in their feelings towards math over time. For some, they loved math as a child but had a change of heart after some particularly “traumatic” experience. The others disliked math until they encountered the right teacher or topic. Many students spoke about the stark reality they encountered in the later years of school: once they found out math was not just memorization:

I liked math as a kid (it was easy for me to memorize things). It wasn’t until I got older and began learning more complex math skills that required less memorization and more thought that my feelings about math began to shift from “it’s great!” to “I’m not really a math person.”

Many shifts originated from interactions with teachers. Some students attribute their teacher’s lack of training to their disinterest in math.

As a youngster I used to enjoy math; however when I reached 9th grade, all that changed. I was placed with a teacher who herself had no knowledge of math; she didn’t know how to explain clearly to us, and so that effected my performance, and in return affected my self esteem, which led to my disliking math.

Another student wrote:

My most salient math experience occurred in approximately 6th grade during long division instruction. I was confused about a particular step in the process and asked my teacher, Mr. X for help. Rather than explain what I needed to do to correctly finish the problem, he said (and I quote) “Kelly, I have already explained this. You must be stupid.” It was that defining moment in which I decided that I hate math and, essentially, forever blocked any interest in seriously attempting to understand or learn mathematics in any way, shape, or form. After all, my math teacher told me I was stupid, so it must have been true. To this day, I get nervous if I have to do any simple math without the aid of a calculator!

In stark contrast to Mr. X, some teachers made equally deep, yet far more positive impressions on their students:

I moved from an Arabic based school to an English school and changing numbers and writing from one direction to another did not help. I remember my teachers giving me a high enough grade just so I can pass because they did not want to deal with me in their classes for an extra year. This happened until I was in high school. There my math teacher Mr. Y did not accept me being a student who was pitied but he wanted me to learn that math and numbers are not so scary. He spent more time on chapters rather then flying past them just so he could finish the curriculum. He taught me algebra and through his classes I learned to like numbers instead of hating them. He also said that we did not need to memorize for hours that math was like a game, it was fun. Because of him I was more interested in math and I wanted to solve problems. He helped me realize that facing numbers will make me come over my fear of math.

Behind many of these reflections was a teacher, parent, or classroom activity that
Excited the student’s interest in math or created a barrier between them and the subject. Unfortunately, students who mentioned their parents did so to illustrate negative experience. No matter how well intended, not all parents can explain things clearly to their young child.

I remember frequently I would not understand a math concept taught in school. I would ask my dad for help with homework when I got home, but he’s a research physicist (must have missed that gene!) and while he meant well, I would easily grow frustrated when he couldn’t always bring it down to my level. I enjoyed math when I understood it, but it was often difficult to get to that point.

And,

Math has always been a dreaded subject for me. I can recall sitting at the kitchen table with my dad as he tried to explain my math homework to me. He wasn’t all that patient, especially since he was an accountant and understood everything perfectly and didn’t understand how or why I was having problems grasping things . . . Since then I have yet to think of math in a positive way. In fact I have to take a math course for my New York certification, and I’m trying my hardest to find a way around it.

Many students also spoke of the competitive nature in their elementary school mathematics classes. For some this was something on which they thrived: speed tests of the multiplication tables, playing “around the world” with multiplication problems, working towards the next, more challenging worksheet. Being the fastest meant being the best and the smartest, and “everyone” in the classroom knew who that person was.

My most salient experience when I was learning math was during the second grade when we had to do a sheet of 30 multiplication problems as quickly as we could right after lunch everyday. I loved it because I think I was pretty good at it and because it was super competitive. You had to finish in under the allotted time and get all of them right to move to a harder sheet the next day and everyone in the class knew what level of multiplication tables everyone else was on and how fast they did it.

Other students found that the competitive nature of the classroom turned them off to the subject completely.

Each day the teacher would hand out a ditto with approximately 50 multiplication tables. We were to have them face down and when the teacher would yell, “Go” we were to flip over the paper and begin furiously answering as many as we could in 20 seconds. At the end of the exercise, we were to hold up our papers so that the entire class could see how many questions we answered on the page. Math to me has always been my most humbling subject.

Many students mentioned feeling shameful, inadequate, or embarrassed because of their perceived inability to “get” math. One student wrote:

I remember our teacher telling us ‘subtract’ and I had no idea what he was talking about. I knew ‘plus’ and ‘minus,’ but I had never heard the word ‘subtract’ before. I vividly remember feeling embarrassed, scared, and inadequate...

Another student echoed this sentiment,
I recall that when I asked the teacher for help, I was usually told to refer to the examples in the textbook and to check my answers using the key at the back of the book. It was horrible. I remember feeling frustrated and inadequate.

It is clear from reading all 127 salient math memories that whether it was the classroom environment, having one terrible or one great teacher, being placed in the wrong math level, or having a well-intended parent, the origins of strongly held emotions towards math vary but the outcomes are the same: from an early point students feel categorized as being a “math person” or “not a math person” -- something that for many dictates the rest of the math career in school. One student wrote, “I feel like I was unfairly relegated to becoming a ‘non-math person’ when I had the potential to be equally as spectacular in math as I am in other subjects.”

**Reflection 2: Math is all around us/play**

Most preservice ECE students have not had the opportunity to see how capable young children are of understanding and learning math concepts. Once videos of children “in action” are introduced to the class, lively discussions ensue. Students often experience an “aha” moment after watching and talking about the various clips. They begin to see that children are not such concrete learners, as they had previously thought. Based on their own observations, they come to realize children are far more capable of exhibiting mathematical behaviors than the general population would expect.

One exploratory study asked 38 students from the Fall 2008 semester of the DMT course about their expectancies before and after completing their final Clinical Interview project (Hartman, 2010). The students were presented with 10 math activities representative of what a typical preschool child is capable of being introduced to: shapes, patterns, counting, number recognition, spatial awareness, etc. For each activity they were asked, “At what age you think a typically developing 4-year old would be ready to start learning this topic?” Students selected ages from 2 to 6 at .5 year intervals. The results showed that students thought children would be able to start learning the material at a younger age ($p<.05$) than they had thought prior to conducting their interview (Appendix A: Change in Teacher Expectancies). This was the case with 9 of the 10 tasks presented.

Results such as these are evidenced by student reactions to the “Math is all around us/play” session of the course. Most watch in disbelief as children demonstrate mathematical thinking during time in the “dramatic play” area of the classroom or in the block corner. The idea that children this young are already thinking mathematically is something most people are not aware of. Typically, this class session opens the students’ eyes for the rest of the semester.

I found it interesting to see math in things I would not normally relate to math. For example the block play, of course there is mathematical thinking required in this type of play, but I had never really seen it like that. This week’s topic has inspired me to be more analytical in my observation of play through a mathematical lens. I am curious to see where else I will find math in young children’s play where I may not have noticed it before.

Another student reflected on her reaction to a video seen during class.
From the videos, especially of the girl with the scale and the boy who was lining up the numbers written on different colored shapes...there was no teacher in sight, but yet they were learning on their own, or going over what they learned by themselves. They were testing out their own waters, which really put a smile on my face.

Similarly, the fact that math is everywhere in the students’ OWN world is something they are not conscious of either. During this class session students are made aware of the math they carryout daily. Whether it be measuring coffee for the coffee maker, reheating something in the microwave, writing a check, or figuring out what time they will be getting home, the students begin to see that math is truly all around them.

I had not realized that math is so much a part of our world. Now I see math everywhere. Even in the precise placement of the keys on this keyboard I’m using. For example, on the train there is a grouping of 8 seats and then a space, a grouping of 8 seats and then a space, and so on.

This course session serves as a major turning point for the students with regards to their openness to the course content. We see this in their reflections through expressions of surprise, wonderment, and excitement at the idea that children are already thinking mathematically, and that math can be found everywhere. This discovery affects the way they receive the rest of the information delivered by the course, and of course, how they view children everywhere.

Until I took this class, I did not realize how children often discover math principles and patterns on their own during free play. Now, when I interact with the children that I work with everyday, I try to capitalize on their natural abilities and offer them challenges to help them understand what it means when they do certain things (e.g., putting colored blocks in a pattern)

Reflection 3: Transition to Symbols

This session introduces students to formal mathematical concepts such as written numbers and abstract mathematical symbols. The major theme that emerged from the reflections was that preservice teachers do not realize how difficult it is for young children to understand written numbers and abstract symbols. As adults, these symbols are part of our everyday life--they seem “easy” to understand because we use them regularly. Mathematical symbols, however, stand as an obstacle for many children during the transition to formal mathematics. To a child, these symbols appear arbitrary and look like a foreign language.

Among the symbols most frequently and hotly discussed is the equals sign. One student wrote, “I was amazed to find out that kids do not fully understand what the equals sign means, even when the teacher is conscious of teaching the concept.” At its conceptual core, the equals sign is very complex to understand and therefore difficult to teach to young students. Very few reflections show that the students fully grasp the meaning of the symbols. One wrote:

I thought the topic on symbols was very interesting, since I never really think about these things myself, and if I’m not consciously aware of these things, how will I relay them to my students? The fact that some children did not know that the = sign means “equal to” or “the same as” is kind of scary!! I look forward to learning more about the kinds of things kids are learning but not REALLY learning so that I will be able to focus on those things and ensure the children really
Another student wrote:

I didn’t completely understand the harm in children defining the equal sign with “makes.” I know that an equal sign means both sides are the same, but, when used in a number sentence such as 5+4=9, isn’t it accurate to say five plus four makes 9? I thought it was arguable that because a child used the word “makes” instead of “equals” is an indicator that the child does not understand the meaning of the symbol.

Even when the teacher does understand the symbols, many are unsure of how to present them to their students.

I really am beginning to understand why it is so difficult for young children to move from the concrete to the symbolic. It was also very profound to learn that many young children who can add and subtract using symbols still don’t know what the equal sign really means and are unable to transfer it to a reverse problem because they never see that. For example many of the children knew that 2+3=5 but they did not think it would be true that 5=2+3 because their teachers never explained to them the concept behind the equal sign.

Research tells us that young children find it easier to follow along with a math story than to follow a formal math sentence representing the same information. Transforming the information from a math story (e.g. “Sally ate two cupcakes and John ate three. How many did Sally and John eat all together?”) to a math sentence (2+3=__) is difficult to do without prior instruction on the special math language. Direct instruction in forming these math sentences guides children down the proper path to linking the stories to their symbolic equation. The difficulty for preservice teachers is in deciding how to deliver this instruction. There are arguments over using rote memorization as an instructional technique.

I attended elementary school in Taiwan, where math is an important part of the curriculum. I grew up being taught that drills would improve my math skills, and while drills were not always fun, they were rewarding. It was rewarding to understand math problems and get them correct. I was able to see my progress as problems became more difficult and the time it took me to answer them correctly shortened. Because of my abilities and the almost instant gratification of correctly answering math problems, math was my favorite subject all throughout elementary school. It was so much my favorite that I even enjoyed filling out pages in drill books!

Other students questioned whether drill and practice actually help relay the information at all. While some students are passionately against the idea of “drill and practice,” we know that many children enjoy such activities. Though this approach can result in memorization, it does not necessarily promote real understanding.

I grew up loving math - I loved numbers and loved how all the problems had an answer. I breezed through math lessons in school and never once thought about whether the lessons were “good” or not. I loved testing my memory and being able to “do” math in my head. But I was never good at word problems. I struggled with the literacy behind the math - the math words and the wording of the problems always caused confusion; I didn’t know how to translate the words into numbers. After writing a paper in college, I realized that my teachers never properly taught us math vocabulary or had math conversations with us.
The transition from informal to formal mathematics is often challenging to both the student and the teacher. This course session excites students’ thinking about formal schooling because they are now thinking about how the children, whom they have just learned are more capable than previously thought, will integrate this information into what they already know. How to introduce topics that children have never seen before is at the core of this topic. And, as we have read, people have differing ideas on how and when this introduction should be made.

**Reflection 4: Curriculum and teaching**

According to the Preschool Curriculum Evaluation Research study (Preschool Curriculum Evaluation Research Consortium, 2008), most ECE programs do not include any math-focused time or other formalized math experiences. Instead, most programs rely on integrated math experiences in which mathematics is a secondary goal and often incidental. Since we know that ECE teachers’ ideas of children’s mathematical abilities are vague at best, the idea of implementing curriculum does not mix well with their views of “developmentally appropriate early childhood education.” Prior to taking the DMT course, many students have not even considered the idea of implementing a formal curriculum in ECE. Many found it comforting to learn that appropriate curricula do exist for teachers to implement in early education classrooms.

I was beginning to worry that although we have been discussing math topics, related task activities and what children should know and learn throughout their early years, I felt that I had no idea how to educate children on these different concepts over the course of a school year.

Another student remarked,

This week I found it interesting how a curriculum was created. I never saw a chart with the weeks on top and the activities on the side, and “x”s indicating what is supposed to be taught when. I found it so surprising that so much goes into a curriculum, and it is so systematic.

One curriculum students are introduced to during the course is Big Math for Little Kids (BMLK). The idea behind this curriculum is that children can and do use informal skills and ideas relating to number, shape, and pattern while playing with blocks or reading stories. An “everyday mathematics” curriculum like BMLK can provide children with the cognitive foundation for their playtime, as well as other parts of their day. It has been found that spontaneous play does contain explicit math content: children truly enjoy exploring numbers and patterns. We see this even when they play with clay or at the “water station” in their classroom. An everyday mathematics curriculum like BMLK provides the teachers with guidelines for creating goals, developing content, and understanding the nature of early childhood mathematics education. As Ginsburg put it, “If we create and employ a challenging and playful mathematics curriculum, then...play can indeed produce learning—even mathematics learning” (2006). For many though, “play” and “math” go together like oil and water. During the course of the semester students begin to see that a marriage of the two is a possibility.

I am currently enrolled in a play course at Teachers’ College, and I think it would be extremely interesting to hold a debate between our courses, especially because today we discussed
a play versus a more structured curriculum. I am a strong believer in a mixture of the two, that is, until this semester and my play class. I have learned so much about play that I never even thought of before. Play is so much more important than anyone would ever believe it to be, especially because play can be a foundation for emotional, physical, and social development. The same I believe applies to a structured curriculum, to a certain extent. I believe that students can learn a great deal in a structured environment and curriculum, both emotionally and physically, but I believe that play is more important in developing social skills inside a classroom.

You can integrate play into a curriculum, but I have learned that it is essential for students to be able to structure their own play environment with their peers during their free time in school.

One student concluded after the session that in they were starting to see that “children can have guidance and be free spirited” while learning math.

Self-efficacy

Our students sometimes revealed doubts about their own self-efficacy in teaching mathematics. Bandura (1997) defines self-efficacy as being a person’s self-perception of competence as opposed to their acceptance of their actual level of competence. “The self assurance with which people approach and manage difficult tasks determines whether they make good or poor use of their capabilities . . . Insidious self-doubts can easily override the best of skills” (p. 35). Based on the students’ reflections, the DMT course could help increase students’ self-efficacy.

In a separate study, using 38 students from Fall 2008 DMT course, preservice teachers’ efficacy for teaching mathematics was measured before and after completing their clinical interview (CI) project (Rosenfeld, 2010). The hypothesis was that conducting the CI would increase preservice teachers’ efficacy for teaching math. The Teachers’ Sense of Efficacy Scale, developed by Tschannen-Moran and Hoy (2001), and the Teacher Efficacy Scale, developed by Gibson and Dembo (1984) were selected due to reliability and validity meeting recommended social science standards, including convergent and discriminant validity (Tschannen-Moran & Hoy, 2001), justifying their use in future studies. It was found that personal teaching efficacy did improve significantly (t=-2.113, df=37, p<.05), and that this change was small to medium in size (d=-0.31). Also, teachers did view math as significantly more age appropriate after conducting and analyzing their CI (t=-12.87, df=37, p<.001, with a large change of d=-4.00).

Cultural differences

The second emergent theme dealt with the perception of cultural differences and the reality of those differences. About 15% of the students indicated they had spent some portion of their life living outside of the United States. Students with experiences in different cultures tended to have beliefs about mathematics education that differed from American students’. Generally speaking, students from Asian backgrounds were not against the “drill and practice” method. They saw this exertion of effort as being necessary to successfully learning mathematics. Other students, as we have read above, felt memorization resulting from drill and practice does not lead to understanding. One student remarked on the cultural differences:
I found it interesting that differences existed in not only students’ math experiences but also their viewpoints on each other’s math experiences. Culture may play a big role in shaping one’s viewpoints; for example, some students thought loving drills was horrible but in the Asian culture, it’s very acceptable, even encouraged.

Another student pointed out that the idea of play and learning are considered very, very different activities:

I grew up in a very strict and structured educational setting. I do not remember much of my pre-school experience but I definitely know that there was not much of ‘play’ in the classroom except during snack break, during which, all the kids went outside to take a break from the classwork. In Ghana, where I am from, play and learning are seen as two very different things, and cannot be integrated. The lesson and readings on Everyday Mathematics, and about the role of play in Mathematics is all very new and interesting to me. I would never have thought to see the mathematical side in the play of young children, or to seize upon the mathematical concepts in play as further opportunities to teach or learn.

These emerging themes show us how complex learning to teach early childhood mathematics can be. When you take into account each person’s unique and oftentimes very personal experience with math, where they are from and what worked best for them when they were students, it becomes clear why there is so much confusion regarding the implementation of mathematics in ECE.

Conclusion

Without understanding our teachers and improving how they are trained to teach, it will continue to be difficult to implement change at the classroom level. As our studies show, prospective teachers have encountered many obstacles when it comes to math, both personally and professionally. For some, these obstacles can be so great that they negatively affect how they teach and therefore their students’ success. Further, many prospective teachers have negative views about children’s abilities to learn math and the desirability of teaching it.

The DMT course encourages the students to think about their teaching and children’s learning through careful observation and analysis. The course helps to change their views of children’s competence and what early mathematics education is all about. It is clear from what we have read in our students’ reflections that a course along the lines of ours can be beneficial for prospective ECE teachers, as this student states:

Overall, this course has significantly impacted my view of early childhood math education. Prior to my enrollment in this course, I was under the impression that a rudimentary math foundation was sufficient for children in pre-school. I now know the complete opposite is true and that math can be taught in a way that is both developmentally appropriate and challenging . . . I especially hope to implement the different methods of assessment, which include observation, testing, and clinical interviews in my teaching practice. This course has illustrated these methods to be extremely helpful in gaining insight and further understanding of children’s mathematical thought processes.
References


RAND Mathematics Study Panel, & Ball, D. L., Chair. (2003). Mathematical proficiency for all students: toward a strategic research and development program


Appendix A: Change in Teacher Expectancies

<table>
<thead>
<tr>
<th>Ready to Learn</th>
<th>Mean Age Pre</th>
<th>Mean Age Post</th>
<th>Paired Sample t-test</th>
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<tbody>
<tr>
<td>Q1 Shape Attributes</td>
<td>3.38</td>
<td>2.94</td>
<td>t=2.45 df=33 p&lt;.020*</td>
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<tr>
<td>Q2 Subitizing</td>
<td>4.29</td>
<td>4.12</td>
<td>t=.620 df=33 p&lt;.540</td>
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<td>Q3 Number Sense</td>
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<td>3.12</td>
<td>t=1.876 df=32 p&lt;.070</td>
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<tr>
<td>Q4 Concrete # Addition</td>
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<td>3.85</td>
<td>t=3.34 df=32 p&lt;.002*</td>
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<td>4.39</td>
<td>3.77</td>
<td>t=3.868 df=30 p&lt;.001*</td>
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<tr>
<td>Q6 Bag It</td>
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<td>3.12</td>
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<td>3.70</td>
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<td>Q8 Small # Comparison</td>
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<td>3.58</td>
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<td>Q9 Cardinality</td>
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<td>3.36</td>
<td>t=4.822 df=32 p&lt;.000*</td>
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<td>Q10 Spatial Relations</td>
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<td>3.53</td>
<td>t=2.701 df=32 p&lt;.011*</td>
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*significant at the p< .05 level