Learning to Think About Early Childhood Mathematics Education: A Course

Herbert P. Ginsburg, Susan Jang, Michael Preston, David VanEsselstyn, Ayelet Appel

Summary

The recent and welcome proliferation of preschool programs throughout the country has created an increased need for qualified teachers of early mathematics. Yet universities and colleges offer little instruction in early childhood mathematics education. As a result, many practicing and prospective daycare providers and teachers do not receive the training they need to promote early childhood mathematics. This paper describes a course designed to prepare college and graduate students (including practicing teachers) to teach mathematics effectively to young children. The course introduces students to a comprehensive developmental approach to mathematics education, including the psychology of mathematical thinking and learning; methods for observing, interviewing, and evaluating children; sample mathematics curricula; basic ideas of mathematics; and principles of early childhood pedagogy. The course makes heavy use of video and web-based technology to help students integrate the research literature with their own observations to develop personal yet disciplined theories that can guide teaching in practical ways. The course is intended to serve as a model for college educators and to stimulate the rethinking of current approaches to the preparation of early childhood teachers.

Why We Need the Course

Three factors have led to a veritable revolution in early childhood education that is now unfolding across the United States. One is that children from China, Japan, and Korea outperform their American counterparts in mathematics achievement perhaps as early as kindergarten *(Stevenson, Lee, & Stigler, 1986)* and certainly by first *(Stevenson, Lee, Chen, Lummis, Stigler, Fan, & Ge, 1990)* and then fourth grade *(U.S. Department of Education, National Center for Education Statistics, 1997)*. A second factor is that low-SES children—a group comprised of a disproportionate number of African Americans and Latinos *(National Center for Children in Poverty, 1996)*—show lower average levels of academic achievement than do their middle- and upper-SES peers *(Natriello, McDill, & Pallas, 1990)*. A third and more promising factor is research indicating that preschool children already possess a surprisingly competent informal mathematics *(Ginsburg, Klein, & Starkey, 1998)*, are ready to learn complex mathematics *(Greenes, 1999)*, and that a strong foundation in preschool education can promote learning in later years *(Bowman, Donovan, & Burns, 2001)*.

In response to the educational need and opportunity, states like Texas are expanding preschool programs, particularly for disadvantaged children. Georgia and New York have adopted a policy of universal preschool education. Educators have come to realize that mathematics needs to occupy a central place in early childhood education. The National Council of Teachers of Mathematics and the National Association for the Education of Young Children have collaborated to produce a joint position statement advocating increased attention to early childhood mathematics education *(National Association for the Education of Young Children & National Council of Teachers of Mathematics, 2002)*. As a result of all these events, many teachers and prospective teachers across the U.S. are now faced with a mandate to teach mathematics to four- and five-year-old children (preschool and kindergarten children).

Many teachers and prospective teachers across the U.S. are now faced with a mandate to teach mathematics to 4- and 5-year-old children.

Most teachers and prospective teachers, however, are poorly prepared to undertake the task. At the college level, courses in teaching early childhood mathematics are rare. For many years, the early childhood community believed that organized mathematics education is not necessary or is even undesirable for young children; hence there was no need for a course in early childhood education. In addition, schools of education have downplayed the importance of teaching mathematics, even at the elementary school level. As a result, education students are required to take many reading and pedagogy courses, but only one "math methods" course.

Whatever the reasons for its virtual absence, a course in early childhood mathematics education is now essential for undergraduates who are preparing to enter a teaching career; graduate students who seek early childhood certification; junior college or community college students who wish to obtain an associate's degree; and teachers or daycare providers who need course credits in order to maintain or improve their professional status and raise their salary.

This paper describes a course that we believe can help prepare students to conduct effective mathematics education for children from preschool (age 3 or 4) through roughly the first grade. We discuss the goals, methods, and content of a course, and suggest modifications that may be necessary for different audiences. We hope that the course will provide a model for college and university educators and will stimulate the rethinking of current approaches.

Course Goals

Our main goal is to help the prospective teacher to think deeply about early childhood mathematics and to adopt a comprehensive developmental approach to mathematics education.

Our main goal is to help the prospective teacher to think deeply about early childhood mathematics and to adopt a comprehensive developmental approach to mathematics education. Several types of knowledge and skill are crucial, and our course is structured to help students learn them. The teacher needs to acquire expertise in several areas, including, but not limited to the following:

- To understand what children already know about mathematics, how they think about it, and how they learn it. Research shows that young children have surprising interests and competence in early mathematics *(Ginsburg & Baron, 1993)*.

- To observe, interview, and test in order to assess and interpret what children know, how they think, and what they have learned. Assessing and understanding individual children is essential for responsive teaching *(Bowman et al., 2001)*.

- To know and appreciate the mathematics that children must learn. Early education deals with big mathematical ideas, like cardinal number and pattern, and teachers need to understand them *(Ma, 1999)*.

- To develop practical, personal, and disciplined theories of children's mathematics education. As psychologist and philosopher William James pointed out, education is always filtered through the teacher's "intermediary inventive mind" *(James, 1958, p. 23)*. The teacher has to make her own sense of children and how they learn.

- To use effective pedagogy. The teacher needs to understand, develop, and use sound principles for teaching young children *(Lampert, 2001)*.

- To understand how to evaluate early childhood mathematics programs. A college course cannot fully prepare teachers to implement a particular program. Instead, the course should help teachers to think about programs and how to implement them effectively.

The Overall Structure

We assume a 15-week course (the *weeks* being designated in Column 1 of Table 1), with each week comprising three 50-minute sessions. The second column depicts the *content* of the course, week by week. Thus, after an introductory session, weeks 1 through 9 mostly include material relating to the psychology of the child's mathematical knowledge. The second strand in the content sequence, weeks 10 to 14, examines pedagogy and curriculum. The last week provides students with opportunities to report on their work and reflect on what has been learned.

The course does not present mathematics as a separate topic. Instead, mathematical concepts are introduced as an integral part of all topics covered. For example, during week 2, students examine children's learning of the counting numbers. This examination must necessarily entail a mathematical understanding of counting itself. Thus students learn that a mathematical pattern—the base ten structure—underlies the counting numbers. Without knowledge of this mathematical idea, students cannot teach counting effectively (for example, by stressing the key elements of the base ten pattern rather than rote memorization) and cannot understand children's counting mistakes (like "twenty ten," which is the child's attempt to produce a pattern-based counting number).

The third column in Table 1 refers to *videos* used to illustrate key concepts. Thus, in week 1, students encounter a video clip of a baby doing natural mathematics as she plays with her mother, and in week 10 the students analyze a tape of a classroom lesson on mapping.

The fourth column in the Table refers to students' learning two major *methods*, observation and clinical interviewing, and applying them to a final project. In roughly the first third of the course, students work outside of class—usually online at home or in a laboratory—on exercises that involve viewing and interpreting videotaped examples of

Table 1. Organization of Course

Week	Content	Example of Videos	Methods	Assignments
1	Introduction; Baby mathematics; Basic concepts	Block play; Baby with rings		
2	Counting words, enumeration, and cardinal number	Boy identifying counting mistakes	Observation	Short assignment
3	Mathematics in everyday activities	Children creating patterns	Observation	
4	Transition to symbolic mathematics	Child learning the = sign	Observation	Short assignment
5	Clinical interviewing and assessment	Extended interview	Observation	
6	Number facts	Child determining whether a number fact is correct	Clinical interview	Short assignment
7	Calculation	Child solving a written multiplication problem	Clinical interview	
8	Understanding	Explaining an answer to a calculational problem	Clinical interview	Short assignment
9	Shape, Space, and Pattern	Children engaged in block construction	Clinical interview	
10	Pedagogy: Constructivism and Manipulatives	Teaching mapping		Final project
11	Curriculum: Big Math for Little Kids	Selected activities		Final project
12	Curriculum: Computers and story books	A computer program and children reading a book		Final project
13	Curriculum: Everyday Mathematics	Selected activities		Final project
14	Curriculum: A textbook	A child's understanding of a textbook page		Final project
15	Presentations and reflections			

mathematical behavior. In the second third of the course, the video exercises shift to clinical interviewing.

The fifth column of the Table refers to *assignments*—both short assignments and the final project. From weeks 2 through 8, students complete short assignments involving analysis of the readings and relevant videos. From weeks 10 through 14, students work on a final project requiring a synthesis of everything learned during the course: they write a paper showing how they used observation and clinical interview to examine children's classroom learning.

Content

Part I: Psychology of Mathematical Thinking and Learning

Week 1 begins by giving an introduction to and a framework for the course. After covering course mechanics, we ask students to analyze a video of young children playing with blocks. The students' task is to identify the kinds of mathematical thinking in which children engage during this activity. Our goal is to have students discover young children's remarkable mathematical interests and abilities. We then introduce a discussion of the history of early mathematics education and consider how our understanding of young children's mathematical knowledge has shaped the mathematics we teach *(Balfanz, 1999)*.

Our goal is to have students discover young children's remarkable mathematical interests and abilities.

Week 1 continues by covering the early development of mathematical concepts such as more, less, and equivalence. We consider how infants and young children are biologically prepared to construct basic mathematical ideas through interacting with ordinary environments that provide stimulating experiences in number, shape, pattern, and space. Environments rich in mathematics enrich the children who live in them *(Ginsburg, 1989, ch. 1)*.

Week 2 addresses the topics of counting and cardinal numbers. From a mathematical point of view, we examine the nature of counting numbers, enumeration, and cardinality. From a psychological point of view, we examine children's rules for generating the counting words, their strategies for counting objects, and their understanding of cardinality *(Ginsburg, 1989, ch. 2)*. Combining the two perspectives helps teachers to understand children's competence and to devise classroom activities.

Week 3 covers different aspects of everyday mathematics. We show how young children engage in a variety of mathematical explorations and applications during free play, some of which may be considered quite advanced. Children spontaneously investigate patterns and shapes, compare magnitudes, and enumerate *(Ginsburg, Inoue, & Seo, 1999)*. We show how they develop concrete and mental methods for addition and subtraction *(Ginsburg, 1989, ch. 3)*. Appreciating everyday mathematics helps teachers to understand what *developmentally appropriate* instruction should entail.

Week 4 addresses young children's transition from everyday, largely informal mathematics (usually developed spontaneously, without much adult assistance) to the formal, symbolic mathematics introduced in school. We consider children's learning of written arithmetic at school and the difficulties they encounter when trying to learn how to read, write, and understand numerals *(Ginsburg, 1989, ch. 5)*.

Week 5 introduces clinical interviewing and assessment. We provide an overview of assessment issues, and show how clinical interviewing is a powerful method for understanding children's thinking and learning. We consider different ways to implement clinical interviewing in the classroom *(Ginsburg, Jacobs, & Lopez, 1993)*. We also consider current tests and other methods for assessing young children's mathematical knowledge and achievement.

Week 6 covers number facts. We first describe the different ways they can be learned based on understanding and not merely on rote memory. We then discuss how number facts can be taught in meaningful ways *(Ginsburg, 1989, ch. 6)*.

Week 7 introduces the topic of calculation. We uncover the various strategies children use when working on computational problems. We also discuss how examination of children's mistakes can provide insight into their thinking—some mistakes result from systematic strategies stemming from children's attempts to make sense of what they are taught *(Ginsburg, 1989, chs. 7, 8)*.

Week 8 of the course covers the topic of understanding, with particular attention to the idea of constructivism and the use of manipulatives *(Ginsburg, 1989, ch. 9)*. We show that understanding involves connections among various systems of knowledge and skill, and discuss the central role of metacognition. We deconstruct constructivism, presenting the idea as more than an empty slogan. Finally, we discuss the role of manipulatives and how they can be useless if they do not stimulate mental activity *(Clements & McMillen, 1996)*.

Week 9 covers children's understanding of shape, space, and pattern. Although researchers have tended to neglect these topics, they are as basic a part of mathematics as is number. Hence, we review psychological investigations touching on non-numerical aspects of mathematics *(Clements, 1999b; Greenes, 1999)*.

Part II: Pedagogy and Curriculum

Week 10 covers pedagogy. We observe and discuss the knowledge and strategies that are involved in and lead to good teaching. We see how teaching is a complex activity that poses many interesting challenges at the preschool level and beyond *(Ball & Cohen, 2000)*.

The next several weeks introduce students to a variety of curricular approaches and materials. The goal is not to train students to implement any particular curriculum; that is the task of in-service professional development. Instead, we intend to help students think deeply about curriculum issues such as: What makes a curriculum developmentally appropriate? How should curricula be evaluated?

We intend to help students think deeply about curriculum issues such as: What makes a curriculum developmentally appropriate? How should curricula be evaluated?

Week 11 reviews a comprehensive mathematics curriculum for young children, *Big Math for Little Kids* *(Balfanz, Ginsburg, & Greenes, 2003; Ginsburg, Greenes, & Balfanz, 2003)*. Students examine the logic behind the program and consider how its activities draw upon psychological knowledge to teach big mathematical ideas. Students critique selected activities in the areas of number, shape, pattern, measurement, operations, and space.

Week 12 turns students' attention to two very different approaches to teaching mathematics in the early years, specifically computers and storybooks. Students examine how computers can be used effectively with young children *(Clements, 1999a)*, and explore the new Clements and Serama software program designed to teach basic aspects of shape and space *(Clements & Serama, 2003)*. Students also review the role of literature in presenting mathematical concepts *(Hong, 1999)*, with special attention to recent mathematics storybooks *(Casey, Anderson, & Schiro, 2002)*.

Week 13 entails study of the *Everyday Mathematics* curriculum *(University of Chicago School Mathematics Project, 1998)*, an innovative effort that extends throughout elementary school. Students critique the program, examining the extent to which it successfully employs constructivist principles and effectively uses manipulatives.

Week 14 ends the substantive coverage of the course by examining the role of textbooks in early mathematics instruction. Textbooks are widely used in mathematics education, even during the early years, and hence are important to understand. Students critically review a popular textbook at the first-grade level. They examine the extent to which the material draws upon children's informal knowledge, presents material in a coherent manner, and employs useful models of mathematical concepts.

The last week is reserved for student presentations, discussion, and reflection.

Videos

Every class session makes extensive use of videos to illustrate concepts and stimulate student thinking. The videos tend to be brief, sometimes shorter than one minute, but typically no longer than two or three. Videotaped observations of children's free play, classroom teaching episodes, and interviews on children's mathematical concepts provide students with concrete examples of specific concepts covered in the readings. One minute of rich video can generate a good 10 minutes of discussion; in a typical class, the instructor may use 10 or 20 minutes of tape, so that 2 or 3 hours of tape might suffice for a whole semester.

A key feature of our method is the way in which videos are used to provide a kind of lab experience for the students. The videos provide opportunities for students to examine and interpret various aspects of children's behavior and to formulate and refine their own views based on the video evidence and the material that they have read. The videos help students actively process the course content as well as develop their own skills in an authentic context. The tapes do not take the form of an educational television program. Instead, the instructor leads the students in an active discussion of the videos. The instructor shows a short excerpt, asks the students what it means, encourages their interpretation of it, promotes discussion of their interpretations, encourages disagreement among students, and pushes students to cite the evidence employed to arrive at a particular interpretation. Then, the instructor might play the video again, in order to clarify what the child actually said, to re-examine the child's hand movements or facial expression and the like. The students are constantly encouraged to consider the evidence and what it means for their interpretations, which of course can and should be revised and refined as the process of examining video proceeds. Learning of this type is meaningful (it is more than memorization of current theories or research findings); it is constructive (students learn to use evidence to develop their own interpretations); and it is practical (students learn to interpret mathematical behavior so as to improve the teaching of mathematics).

The videos provide opportunities for students to examine and interpret various aspects of children's behavior and to formulate and refine their own views based on the video evidence and the material that they have read.

Methods

In addition to studying content, students work on exercises outside of the classroom to help them learn observational and clinical interview methods.

Observation

From weeks 2 through 5, we help students learn to observe. Our goal is to help students develop their ability to observe learning, thinking, and teaching as they are exhibited in everyday behavior and in learning activities in the classroom. We want students to pay careful attention to children's free play, language, and problem solving during mathematics activities and to any other behaviors that shed light on children's learning and understanding. We also want students to learn to observe

> ***Observation is not simply passive looking. It is active analysis and conceptualization.***

teaching with a careful and critical eye so as to identify key pedagogical strategies and assumptions. In our view, observation is not simply passive looking. It is active analysis and conceptualization. The sensitive observer not only gathers the relevant evidence carefully, but also reasons about it, concluding from the child's placement of one block next to another that she is using an intuitive idea about symmetry or, from her response to the teacher's questions, that she fails to understand the idea of equivalence. In brief, being a good observer involves an objective attitude towards the facts and also a disposition to think carefully about what they mean. Observing is thinking as much as it is seeing.

Our method for teaching observation is to involve students in a series of homework exercises. Outside of the college classroom, the students are asked to view and comment (using online technology described below) upon a series of videos of children's behavior. They are not encouraged to focus merely on superficial aspects of behavior such as "child counts" or "child works with manipulatives." Instead, they are asked to identify key aspects of mathematical learning and thinking that underlie the observed behavior. Eventually, students learn to create their own conceptual schemes for making inferences concerning important dimensions of mathematical learning and thinking.

Clinical Interview

Although it can be useful, observation is often not sufficient for understanding the child's thinking and learning. Sometimes the child's behavior simply does not give enough information to allow the observer to know conclusively what he or she is thinking. Sometimes the child does not exhibit behavior that is relevant to what the observer wants to learn about. Consequently, it is necessary to engage the child in an extended, flexible conversation about the issue of interest—a clinical interview. The student needs to learn what questions to ask, how to engage the child so that he or she freely provides useful information, how to ask questions without suggesting an answer, and how to probe for underlying thought processes. Clinical interviewing takes preparation and time to develop, but can be valuable both in psychological research and educational practice *(Ginsburg, 1997)*. It is an extremely important method for forming, informing, and maintaining the teacher's "intermediary inventive mind" *(James, 1958)*.

> ***Observation is often not sufficient for understanding the child's thinking . . . Consequently, it is necessary to engage the child in an extended, flexible conversation about the issue of interest—a clinical interview.***

Hence, over a four-week period, we engage the students in homework exercises designed to help them learn the techniques of the clinical interview. We ask students to view videos of interviews, to interpret what they show about children's learning and thinking, and to analyze the interviewer's methods of questioning. We ask the students to develop lists of "do's and don'ts" that can guide their own interviewing.

Assignments

Outside of class, students work on both short assignments and a final project.

Short Assignments

On alternate weeks, from weeks 2 through 8, students do short homework assignments outside of class. These require students to express their own ideas and support them with specific video and text citations. Thus, in the fourth week, for example, the student might be asked to discuss what understanding of the equals sign entails, and to cite relevant video episodes that support a particular interpretation. In the technology section below, we describe technology that allows students to embed the video citations within a written essay.

Final Project

Around week 10, by the end of the observation and clinical interview phase, students should have sufficient skill in observation and interviewing to use them in their final projects. They should also have a good appreciation of psychological concepts necessary to understand children's learning and thinking, as well as an understanding of the mathematics children learn on their own and in school. The students' next task is to develop, implement, and evaluate their own lessons or classroom activities based upon the concepts learned in the course. The goal of this final project is to encourage the student to use individual creativity to develop an activity, and then to draw upon and synthesize much of what has been learned in the course—ideas about children's thinking and learning, mathematical concepts, pedagogical principles, and methods of observation and interview—to come to an understanding of how and what children learn from the activity. Hence students not only create an activity but also are required to document their work in essay and/or video format and reflect on its results. They evaluate the activity by observing and analyzing the teaching process, by observing what children seem to learn from the activity, and by interviewing some of the children to discover their learning and thinking processes.

Technology

During the development of this course, we have been working closely with the Columbia Center for New Media Teaching and Learning to create an online learning space that allows students to access course materials and extend their classroom experience in a meaningful, hands-on (or better still, minds-on) way. The key components of the online environment include a *digital library*, and a *digital workspace* in which students can use a *multi-media essay tool*, and *video lessons* in observation and clinical interview.

The key components of the online environment include a digital library, and a digital workspace in which students can use a multi-media essay tool, and video lessons in observation and clinical interview.

The *digital library* is the online space where students can access a collection of course materials, primarily the videos shown in class, electronic forms of various readings, both assigned and optional, and links to journals and websites.

The *digital workspace* provides students with a virtual place in which they can keep and review personal selections from the digital library, including specific episodes they have excerpted from the video clips, and quotations from the readings. The digital workspace also stores the students' short assignments and drafts of the final projects. The workspace contains a kind of electronic portfolio recording the evolution of students' theories about early mathematics education.

The *multi-media essay* tool allows creation of the short assignments and final project. In the latter, students create their own lesson or activity, videotape its use in a classroom, download the video to the online digital workspace, and cite excerpts from the video within the body of the essay. Thus, a student can write about how she introduced a certain lesson and at the appropriate place in the paper cite the two-minute section of videotape illustrating the lesson. By clicking on the citation, the reader can view the segment online. Later in the paper the student can refer to a ten-second segment of one of the class videos to illustrate an observation concerning the teacher's pedagogy, and later still can refer to relevant sections of her clinical interview illustrating what a child learned from the lesson.

The *video lessons* present the observation and interview exercises described above. The observation lessons lead students through a series of screens that prompt them to watch a video clip, comment on it, watch the clip again overlaid with expert commentary, and then reflect on the expert's and their own interpretations. The interviewing lessons similarly prompt students to suggest a question they think should be asked to follow up on a child's interview response, and then compare their question with the one that the interviewer does in fact ask. All of these lessons are done online and outside of the classroom; students learn the observation methods separately from and in parallel with the other course material.

We have recently piloted an experimental version of this multi-media online environment. Techniques like these are not essential to teaching the course, but we think they can make it more stimulating, more effective, and more enjoyable for students and instructors alike.

Modifying the Course for Your Needs

The course described here is intended to be an example or perhaps a template to guide others in creating their own courses. In the near future, we hope to publish guidelines for the course as well as the accompanying videotapes and eventually to make available the multi-media online environment, which we believe can be a powerful tool for stimulating students' thinking and learning. Readings can vary, as can emphasis on particular psychological issues, theories, or curricula. Nevertheless, we believe that any course on early childhood mathematics should attempt, as we have, to present a comprehensive developmental approach that promotes the synthesis of

Any course on early childhood mathematics should attempt, as we have, to present a comprehensive developmental approach that promotes the synthesis of psychological ideas, methods designed to reveal how children think and learn, understanding of the relevant mathematics, principles of pedagogy, and examples of promising mathematical activities.

psychological ideas, methods designed to reveal how children think and learn, understanding of the relevant mathematics, principles of pedagogy, and examples of promising mathematical activities. And our common goal should be to create teachers who can think deeply about early childhood mathematics education.

References

Balfanz, R. (1999). Why do we teach children so little mathematics? Some historical considerations. In J.V. Copley (Ed.), *Mathematics in the early years* (pp. 3–10). Reston, VA: National Council of Teachers of Mathematics.

Balfanz, R., Ginsburg, H.P., & Greenes, C. (2003). The Big Math for Little Kids early childhood mathematics program. *Teaching Children Mathematics*, 9(5), 264–268.

Ball, D.L., & Cohen, D.K. (2000). Challenges of improving instruction: A view from the classroom. In T. Duggan & M. Holmes (Eds.), *Closing the gap: A report on the Wingspread Conference Beyond the Standards Horse Race: Implementation, assessment and accountability, the keys to improving student achievement.* Washington, DC: Council for Basic Education.

Bowman, B.T., Donovan, M.S., & Burns, M.S. (Eds.). (2001). *Eager to learn: Educating our preschoolers.* Washington, DC: National Academy Press.

Casey, B., Anderson, K.L., & Schiro, M., (2002). *Layla discovers secret patterns.* Chicago, IL: The Wright Group/McGraw-Hill.

Clements, D.H. (1999a). The effective use of computers with young children. In J.V. Copley (Ed.), *Mathematics in the early years* (pp. 119–128). Reston, VA: National Council of Teachers of Mathematics.

Clements, D.H. (1999b). Geometric and spatial thinking in young children. In J.V. Copley (Ed.), *Mathematics in the early years* (pp. 66–79). Reston, VA: National Council of Teachers of Mathematics.

Clements, D.H., & McMillen, S. (1996). Rethinking "concrete manipulatives." *Teaching Children Mathematics, 2,* 270–279.

Clements, D.H., & Serama, J. (2003). *DLM Express math resource package.* Columbus, OH: SRA/McGraw-Hill.

Ginsburg, H.P. (1989). *Children's arithmetic: How they learn it and how you teach it* (2nd ed.). Austin, TX: Pro Ed.

Ginsburg, H.P. (1997). *Entering the child's mind: The clinical interview in psychological research and practice.* New York: Cambridge University Press.

Ginsburg, H.P., & Baron, J. (1993). Cognition: Young children's construction of mathematics. In R.J. Jensen (Ed.), *Research ideas for the classroom: Early childhood mathematics* (pp. 3–21). New York: Macmillan Publishing Company.

Ginsburg, H.P., Greenes, C., & Balfanz, R. (2003). *Big math for little kids.* Parsippany, NJ: Dale Seymour Publications.

Ginsburg, H.P., Inoue, N., & Seo, K.-H. (1999). Young children doing mathematics: Observations of everyday activities. In J.V. Copley (Ed.), *Mathematics in the early years* (pp. 88–99). Reston, VA: National Council of Teachers of Mathematics.

Ginsburg, H.P., Jacobs, S.F., & Lopez, L.S. (1993). Assessing mathematical thinking and learning potential. In R.B. Davis & C.S. Maher (Eds.), *Schools, mathematics, and the world of reality* (pp. 237–262). Boston, MA: Allyn and Bacon.

Ginsburg, H.P., Klein, A., & Starkey, P. (1999). The Development of Children's Mathematical Thinking: Connecting Research with Practice. In I. Sigel & A. Renninger (Eds.), *Handbook of Child Psychology: 5th Ed., Vol. 4. Child Psychology and Practice.* (5th Ed., Vol. 4, pp. 401–476). New York: John Wiley & Sons.

Greenes, C. (1999) Ready to learn: Developing young children's mathematical powers. In J.V. Copley (Ed.), *Mathematics in the early years.* (pp. 39–47). Reston, VA: National Council of Teachers of Mathematics.

Hong, H. (1999). Using storybooks to help young children make sense of mathematics. In J.V. Copley (Ed.), *Mathematics in the early years.* Reston, VA: National Council of Teachers of Mathematics.

James, W. (1958). *Talks to teachers on psychology: and to students on some of life's ideals.* New York: W.W. Norton & Company.

Lampert, M. (2001). *Teaching problems and the problems of teaching.* New Haven, CT: Yale University Press.

Ma, L. (1999). *Knowing and teaching elementary mathematics.* Mahwah, NJ: Lawrence Erlbaum Associates, Publishers.

National Association for the Education of Young Children & National Council of Teachers of Mathematics. (2002). *Position statement. Early childhood mathematics: Promoting good beginnings.* Retrieved, from the World Wide Web: www.naeyc.org/resources/position_statements/psmath.htm

National Center for Children in Poverty. (1996). *One in four: America's youngest poor. Abridged Version.* New York: Author.

Natriello, G., McDill, E.L., & Pallas, A.M. (1990). *Schooling disadvantaged children: Racing against catastrophe.* New York: Teachers College Press.

Stevenson, H., Lee, S.S., & Stigler, J. (1986). The mathematics achievement of Chinese, Japanese, and American children. *Science,* 56, 693–699.

Stevenson, H.W., Lee, S.Y., Chen, C., Lummis, M., Stigler, J.W., Fan, L., & Ge, F. (1990). Mathematics achievement of children in China and the United States. *Child Development,* 61, 1053–1066.

U.S. Department of Education. National Center for Education Statistics. (1997). *Pursuing excellemce: A study of U.S. fourth-grade mathematics and science achievement in international context.* Washington, DC: U.S. Government Printing Office.

University of Chicago School Mathematics Project. (1998). *Everyday mathematics.* Chicago, IL: Everyday Learning Corporation.